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EXPERIMENTAL VERIFICATION OF BOUNDED-FORCE CONTROL METHOD

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SUMMARY

An efficient and simple non-linear active control method called Bounded-Force Control (BFC) method has been developed. The theoretical development and application examples of the BFC method has been presented in several publications[1–3] where it was shown that the BFC is more efficient than the Linear Quadratic Regulator (LQR) method in terms of maximum control force and power requirements when applied to an Active Tuned Mass Damper (ATMD) system. Following a brief review of the fundamental concept of the BFC and a discussion of its main advantages, the results of an experimental investigation using a three-storey shear building model excited by a shaking table and controlled by an electromagnetic linear motor are presented. The experimental results were in perfect agreement with the analytical results, thus confirming the validity of the proposed BFC algorithm.

KEY WORDS: bounded-force control; experimental results; active tuned mass damper (ATMD); linear motor; shear building model

INTRODUCTION

One of the main difficulties in realizing the auxiliary-mass type active control systems with the aim of protecting buildings against strong earthquakes is the demand of large control force and power which is usually unrealizable. This condition is especially true if one employs a linear control method such as the LQR-based control methods or any linear feedback methods which usually has predetermined or constant control gains for the whole control process. When large external disturbance attacks the structure, the actuator should supply very large control force to counterbalance the disturbance. An active control system that is designed based on the linear control methods will result in an unreasonably and impractically large actuator to meet the large force capacity demand, which only required at a few instants of time during the control process, while at other times its full capacity will never be utilized. In other words, the design will be uneconomical. The linear way of controlling a structure against highly random disturbances such as strong ground motions and strong winds is considered inefficient. A simple and non-time-consuming non-linear control method which is adaptive in the sense of responding to stochastic characteristics of the disturbance should be developed.

The theoretical development of the Bounded-Force Control (BFC) method with some application examples to the Active Tuned Mass Damper (ATMD) system has been presented in some publications.[1–3] The BFC is a highly non-linear control method which was developed within the framework of the classical optimal control theory by explicitly treated the physical limit of the actuator in terms of control force in its formulation. The resulting optimal control forces composed of rectangular and pulse-like force which was shown to be more efficient than the linearly varying control force as produced by linear control methods. Through comparative analyses it was demonstrated that the BFC is more efficient than the Linear Quadratic

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Regulator (LQR) method in terms of maximum control force and actuator power and this benefit was more pronounced in case of larger earthquake disturbances.

To investigate several practical aspects for implementation of the BFC in an actual building equipped with an ATMD system, an experimental investigation has been performed. After a brief review of the fundamental concept of the BFC, the objectives of the experiment, dynamic characteristics of the model structure and the actuator, experimental set-up, experimental program and experimental results are reported in this paper. It is demonstrated that the experimental results agreed extremely well with the results of analytical simulations.

FUNDAMENTAL CONCEPT

Review of bounded-force control method formulation

Consider a linear time-invariant dynamical system defined in the state-space form as

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \mathbf{C}z(t) \tag{1}$$

where $\mathbf{x}(t)$, $\mathbf{u}(t)$ and $z(t)$ are the state vector, control force vector and external disturbance, respectively. In the case of LQR the control objective defined by the following performance index (PI):

$$\mathbf{J} = \frac{1}{2}\mathbf{x}^{\mathrm{T}}(t_f)\mathbf{P}\mathbf{x}(t_f) + \frac{1}{2}\int_{t_i}^{t_f} [\mathbf{x}^{\mathrm{T}}(t)\mathbf{Q}\mathbf{x}(t) + \mathbf{u}^{\mathrm{T}}(t)\mathbf{R}\mathbf{u}(t)]\,\mathrm{d}t \tag{2}$$

where t_i and t_f are, respectively, the initial and final time of the control process. As is well known, the optimal control force $\mathbf{u}(t)$ will be obtained as a linear function of $\mathbf{x}(t)$,

$$\mathbf{u}(t) = \mathbf{K}\mathbf{x}(t) \tag{3}$$

where $\mathbf{K}$ is the constant control gain matrix for the whole control process. As has been mentioned previously, this linear control law leads to inefficient operation of the actuator capacity.

To increase the efficiency of the actuator operation as well as the control performance, the proposed method is formulated with a modified PI, which excludes the control-effort penalty from equation (2). Besides that, since the actual control process is a discretized process with a relatively short time interval between the response sensing and the control force application, it is more practical to define the modified PI at the end of each time interval as

$$\mathbf{J} = \tfrac{1}{2}\mathbf{x}^{\mathrm{T}}(t_f)\mathbf{P}\mathbf{x}(t_f) \tag{4}$$

To limit the magnitude of applied control force, the control effort penalty is replaced by a force inequality constraint

$$|\mathbf{u}(t)| \leqslant \bar{\mathbf{u}} \tag{5}$$

where $\bar{\mathbf{u}}$ is the force capacity of the actuator. Based on the variational calculus variation, the boundary condition equations are given for the costate vectors at the end of each time interval t_f as

$$\boldsymbol{\lambda}(t_f) = \mathbf{P}\mathbf{x}(t_f) \tag{6}$$

and the optimal control force for each time interval is found as

$$\mathbf{u}^* = -\mathrm{sgn}(\boldsymbol{\lambda}^{\mathrm{T}}\mathbf{B})\bar{\mathbf{u}} \tag{7}$$

where $\boldsymbol{\lambda}$ is the costate vector. The step-by-step solution procedure to obtain the optimal control force was described in detail in Reference 3. The optimal control law (equation (7)) produces rectangular and pulse-like control forces which are very efficient in suppressing seismic-induced vibrations.

In Reference 2, the weighting matrix $\mathbf{P}$ for a SDOF-ATMD system was selected such as the PI represented the total of kinetic and potential energy of the controlled system. However, the employment of this weighting

matrix did not result in much better control performance as compared with the LQR. To improve the performance of the BFC, the following weighting matrix is assigned:

$$\mathbf{P} = \begin{bmatrix} 1 & 0 & p_2 & 0 \\ 0 & 1 & 0 & 1 \\ p_2 & 0 & p_1 & 0 \\ 0 & 1 & 0 & p_3 \end{bmatrix} \tag{8}$$

The existence of coefficient p_2 which takes into account the interaction between the displacement and velocity responses of the structure improves the performance of the BFC significantly. It was shown[3] that for a 10-DOF shear building model equipped with an ATMD system, to achieve equal response reduction with the LQR under the El Centro ground motion of 200 cm/s^2, the BFC only required about one-fourth and one-third of the maximum control force and maximum actuator power required by the LQR, respectively.

To improve the efficiency of the BFC further, the control force magnitude could be partitioned into two or more levels. The selection of appropriate level of control force is carried out on-line by the control algorithm and depends on the level of external disturbance. In case of one-level control force, the acceleration response of the auxiliary mass could remain quite high even though the seismic disturbance level has been subsided. Moreover, for practical application of BFC to suppress structural vibration caused by a wide range of excitation levels, e.g. weak to strong earthquake ground motions, the adoption of one-level control force is inefficient.

Frequency response function

To show the stability of the BFC over a wide range of vibration frequency, the frequency response functions (FRF) of a SDOF-ATMD system under base excitation or external force applied at the primary mass are numerically calculated. The auxiliary mass to structure (primary) mass ratio is 0·01 and the auxiliary mass parameters are tuned to the natural frequency of the structure using the Den Hartog formula developed for conventional TMD.[4] The model is subjected to base acceleration with amplitude $\ddot{z}$ and frequency $\bar{\omega}$, and the applied control force is expressed in terms of F where $F = m_b\ddot{z}$ and m_b is the structure mass. The FRF for displacement response of structure and stroke of auxiliary mass are shown in Figures 1(a) and 1(b), respectively, where ω in the figures denotes the natural frequency of the structure. From these figures, it is clearly seen that the vibration suppression is more significant for excitation frequency equal to and larger than the structure's frequency, while for frequency smaller than the structure's frequency, the vibration suppression is approximately similar to the LQR or other linear control method. It is also clear that the maximum stroke of the auxiliary mass is not linearly increased with the increase of control force magnitude in the frequency region larger than the structure's frequency.

Based on parametric studies, the optimum stiffness and damping of the auxiliary mass are very close to the optimum ones obtained by the Den Hartog formula for passive-TMD system. However, the control performance in case of random excitation is not sensitive to the deviation of stiffness and damping of the auxiliary mass from their optimum values, as will be demonstrated in the next subsection.

Robustness

The robustness of the BFC with respect to system parameter uncertainties was investigated numerically using the 10-DOF model of Reference 3. To take into account the uncertainties in the model parameters, the simulations were carried out by assigning different mass and stiffness coefficients for computing the model response (actual values) and for computing the required control force (assumed values). The simulation results are presented in Table I. In the table, $\bar{\omega}_i$ and ω_i represent the actual (deviated values) and assumed frequencies of the ith mode of the model, respectively. The results presented in the table show that the BFC is very insensitive to both mass and stiffness variations, thus confirming the robustness of the BFC method.

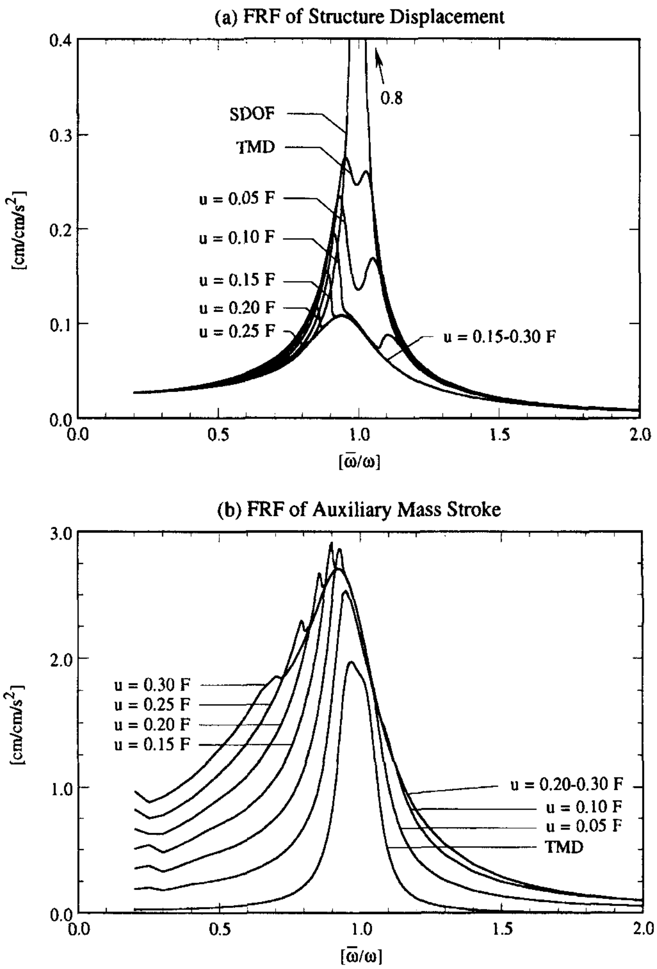


Figure 1. Frequency response functions of SDOF-ATMD system

Table I. Result of sensitivity analysis (robustness of control method)

	$\bar{\omega}_i = \alpha\omega_i$	$\alpha = 0{\cdot}8$	$\alpha = 0{\cdot}9$	$\alpha = 1{\cdot}0$	$\alpha = 1{\cdot}1$	$\alpha = 1{\cdot}2$
Top mass	Rel. disp (cm)	1·67	1·67	1·67	1·67	1·66
	Abs. acc. (cm/s^2)	96·87	97·07	96·93	96·71	96·52
Auxiliary mass	Stroke (cm)	16·69	16·70	16·47	16·24	15·88
	Abs. acc. (cm/s^2)	732·63	695·43	690·92	683·60	675·61
Actuator	u_{max}(kN)	5·00	5·00	5·00	5·00	5·00
	P_{max} (kW)	4·95	4·94	4·93	4·88	4·82

OBJECTIVES OF EXPERIMENT

The experiment was performed with the following objectives.

1. Experimental evaluation of the BFC control performance.
2. Confirmation of the BFC control effect on higher vibration modes, that is, whether the higher mode vibration components are suppressed or excited with the application of control force resulting from the BFC algorithm.

3. Confirmation of the ability of an actuator to supply the rectangular- and pulse-shaped control force to the controlled structure and the effect of non-ideal shape of the control force to the control performance. During the analytical simulations it was assumed that the actuator could produce perfectly the rectangular-shaped control force and the switching between positive and negative control forces could be performed immediately.
4. Investigation of the effect of extending control time interval on the control performance (indirect estimation of the time-delay effect). The results of analytical simulations reported in Reference 1 were obtained under the assumption that there was no time delay between the response measurements and the application of the control force to the structure.
5. Investigation of the effect of reduced feedback quantities (reducing the number of sensors) to the control performance. For actual implementation to building structures, usually only top-storey and auxiliary-mass responses are measured and fed back to the controller.

EXPERIMENTAL SET-UP

To accomplish the aforementioned objectives, a three-storey shear building model was constructed as the model structure as shown in Figure 2. The floors were built of steel plates, while the columns were constructed of laminated rubbers for base isolator. The weight of each storey shown in this figure includes columns and sensors weight, while the third-storey weight also includes the weight of the fixed part of the actuator. The movable part of the actuator that modelled the auxiliary mass weighed about 1 per cent of the total mass of the model.

The block diagram of the experimental set-up is shown in Figure 3. The model structure was mounted on a small-scale shaking table. Sensors measuring the state variables and structural behaviour were attached to the model. The signals from the sensors were directed to a microcomputer and processed by a control program which generated control signals for the actuator. The control force was transmitted to the structure through an electromagnetic linear motor mounted at the centre of the top storey of the model.

Displacement, velocity and acceleration sensors were mounted on the centreline of each storey of the model as well as on the shaking table and on the auxiliary mass. The signals from the displacement and velocity sensors were used as feedback signals to the controller. This information was utilized by the controller to decide the optimal control force for each time interval. The signals from the accelerometers were recorded only for confirming the signals collected by displacement and velocity sensors.

One objective of the current experiment is to investigate the effect of reducing the number of sensors to the top-storey and auxiliary-mass sensors only. To achieve this objective, firstly, several experiments were performed with all-storey feedback and only-top-storey feedback, and the results of both cases were compared. Due to the perfect agreement of the results of both cases, only top-storey and auxiliary-mass responses were then used as the feedback signals.

The on-line computations of the BFC control algorithm was carried out by a microcomputer. The data were transferred to the microprocessor using an analog-to-digital (A/D) converter and the computed data were transmitted to the actuator's driver (amplifier) using a digital-to-analog (D/A) converter.

EXPERIMENTAL PROGRAM

In accordance with the experimental objectives, several series of experiments were performed using the three-storey model structure under the proposed Bounded Force Control. The experimental program, the results of which are detailed in the next section, was as follows:

1. Identification of the dynamic properties of the auxiliary mass (actuator).
2. Identification of the dynamic properties of the model structure.

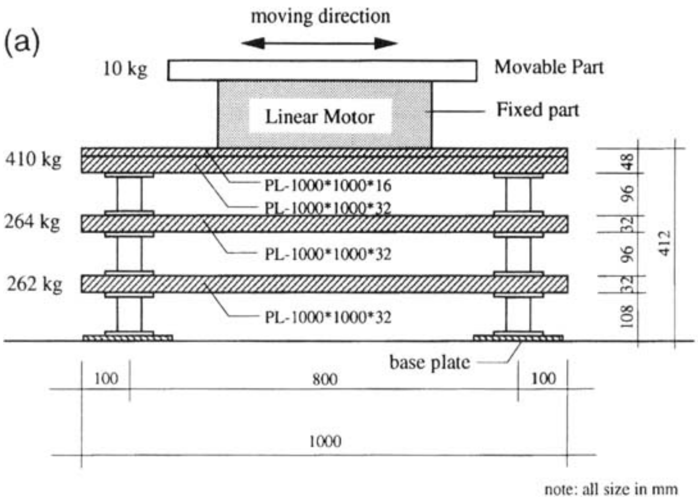


Figure 2(a). Three-storey model structure

(b)

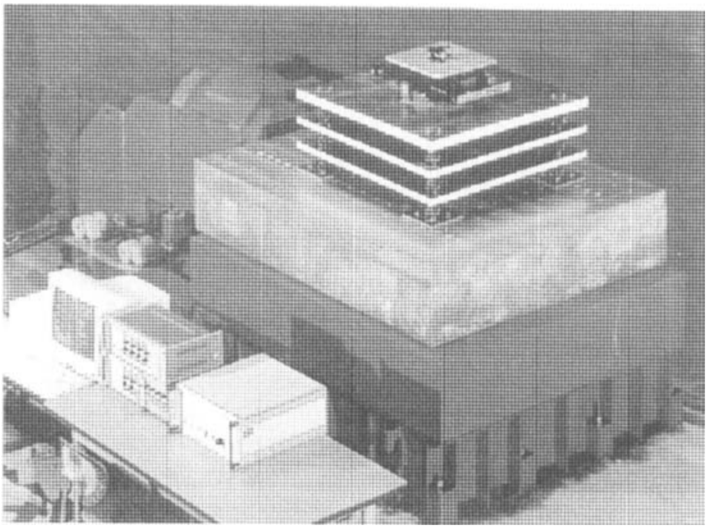


Figure 2(b). Three-storey model structure mounted on a shaking table

3. Active control experiments under:
   3.1. sinusoidal excitations
   3.2. seismic excitations
        Base excitation: El Centro NS and Taft EW ground motion records
        3.2.1 Basic case
              peak acceleration: 15 cm/s^2
              control time interval: 10 ms
        3.2.2 Higher excitation level
              peak acceleration: 30 cm/s^2
              control time interval: 10 ms

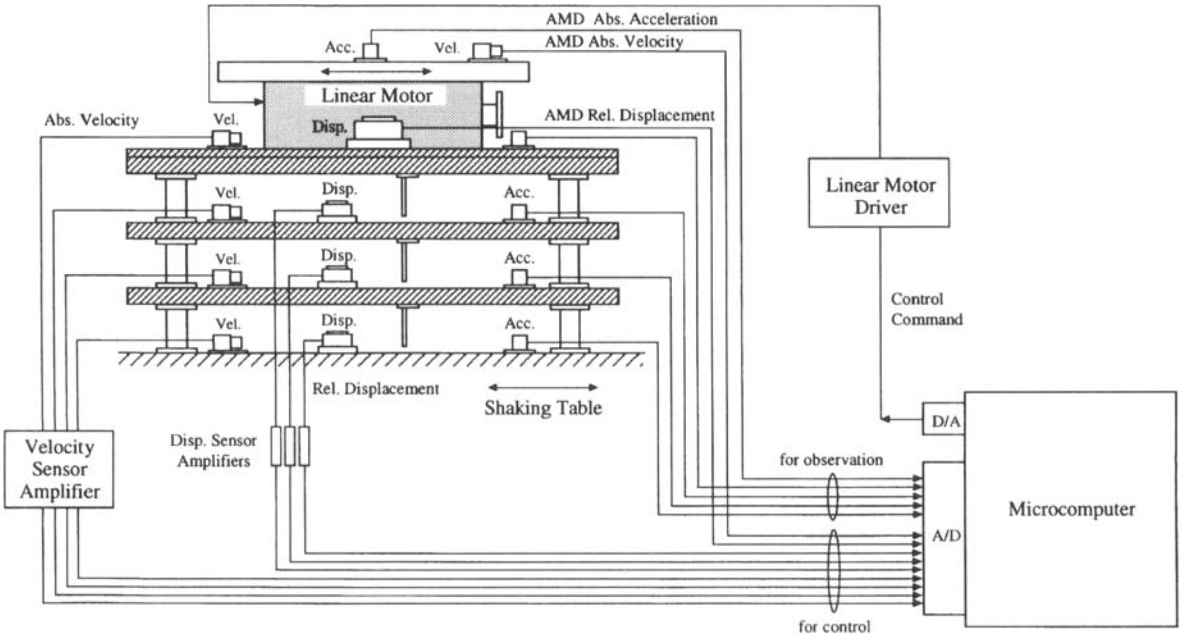


Figure 3. Block diagram of experimental set-up

3.2.3 Longer control time interval
peak acceleration: 15 cm/s^2
control time interval: 30 ms, 40 ms

EXPERIMENTAL RESULTS

Dynamic properties of actuator

For this identification test, a function generator generated and transmitted input sinusoidal signals spanning a wide range of frequencies to the linear motor. From the FFT transform of the response time histories of each frequency level, displacement, velocity and acceleration transfer functions were constructed. The stiffness and equivalent viscous damping constants were obtained by curve-fitting technique applied to the transfer functions as 4·5 N/cm and 1·9 N s/cm, respectively.

The linear motor driver was identified as having high-pass filter characteristics which has undesirable effect on the driver output signals in case of rectangular-shaped input signals as illustrated in Figure 4. It is clear that the driver cannot reproduce the rectangular shape of the input signals; however, the influence of this signal distortion on the performance of the BFC was not significant as will be shown later.

Dynamic properties of the model structure

The natural frequencies of the model structure were determined from forced-vibration tests using the shaking table as the excitation source. The frequencies of the input signals were chosen to span the natural frequency range of the model structure. Frequency response curves were generated from the recorded data of these tests and the modal shapes as well as the natural frequencies were established from these plots. Figure 5 shows the frequency response curves for the model's top-storey acceleration and displacement. The modal damping ratios were evaluated by the half-power method applied to the experimental frequency response curves and verified by curve-fitting technique and found to be as high as 5 per cent for all modes.

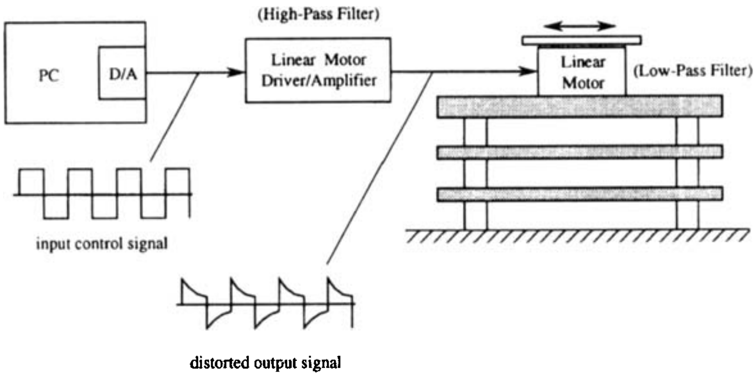


Figure 4. Signal distortion due to high-pass filter

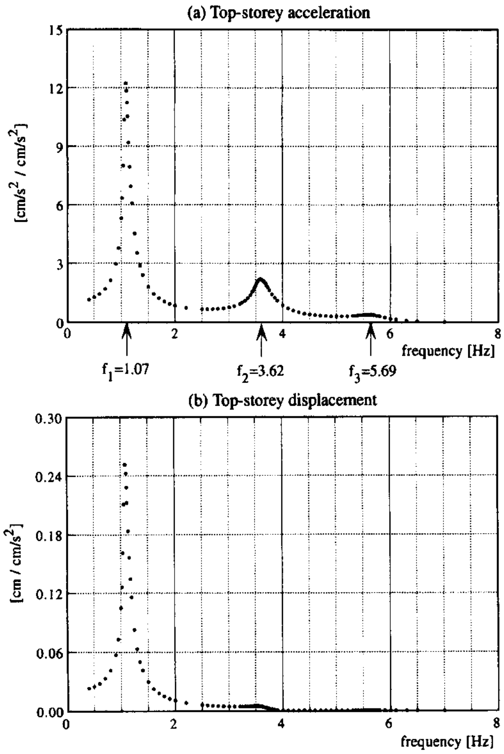


Figure 5. Experimental frequency response curves of model structure

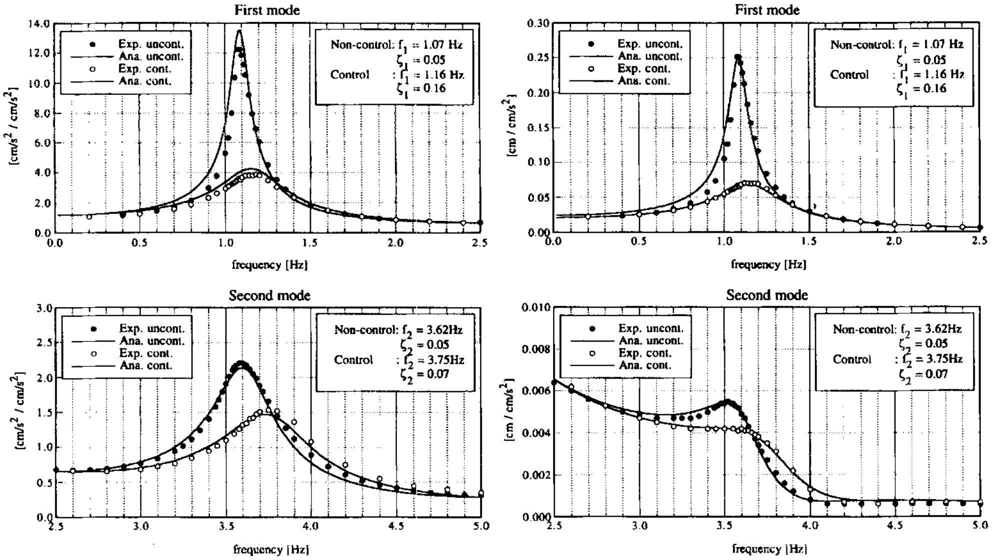
Results of non-control and active control experiments

The damping ratio and stiffness of the auxiliary mass should be actively adjusted during the active control experiments to obtain the desired optimum damping level. The total applied control force is defined as

$$u_{\text{total}} = u_{\text{act}} + u_{\text{aux}} \tag{9}$$

where u_{act} is the active control force obtained by the BFC algorithm and u_{aux} is the control force for adjusting the auxiliary mass stiffness and damping. u_{aux} is defined as

$$u_{\text{aux}} = g_1 x_{\text{aux}} + g_2 \dot{x}_{\text{aux}} \tag{10}$$



(a) Top-storey absolute acceleration

(b) Top-storey relative displacement w.r.t. base

Figure 6. Experimental and analytical frequency response curves of model structure

where x_{aux} and $\dot{x}_{aux}$ are the displacement and velocity responses of the auxiliary mass relative to the top storey and g_1, g_2 are the adjustment factors. For all experimental results presented in this paper, the active control force u_{act} was 10 and 20 N (two-levels control force). These magnitudes of control force were selected to attain the objective of suppressing the top-storey acceleration and displacement responses to about half of the uncontrolled responses under the condition of relatively high first-mode damping ratio (5 per cent) and the limitation of the stroke length of the actuator of only 7 cm.

Sinusoidal excitations. The first active control experiments were designed to quantify approximately the equivalent viscous damping effect that could be produced by the application of Bounded Force Control. To accomplish this goal, a series of active control experiments under sinusoidal base excitations with frequencies covering the whole range of the model natural frequencies were performed. From the FFT transform of the recorded responses obtained during the previous identification test (non-control condition) and the current test, acceleration and displacement FRF curves were constructed as shown in Figures 6(a) and 6(b), respectively. The equivalent viscous damping ratios for the first and second modes were obtained by fitting the analytically obtained curves to the experimental frequency response data. It was found that the first-mode damping ratio could be increased by as much as 11 per cent, while the second-mode damping ratio was increased by 2 per cent. The curve-fitting was applied to the frequency response data of displacement (Figure 6(b)), and the identified parameters were used to construct the analytical frequency response curves for acceleration as shown in Figure 6(a). The analytical curves did not perfectly fit the experimental data due to the stiffness non-linearity of the model structure. The non-linearity was caused by the characteristics of the rubber columns, i.e. during the non-control condition the lateral deformation of the rubber columns were larger than during the control condition which led to higher rubber stiffness during the control experiments. The effect of the model stiffness non-linearity was also made clear by the shifting of the first and second natural frequencies in case of non-control and control conditions.

Further, it was also observed that eventhough the peak at the second mode frequency (see Figure 5) was very low, the control effect on this higher mode was still observable, which implied that the BFC is not only stable in the high-frequency range, but also effective to suppress the higher-mode vibration response.

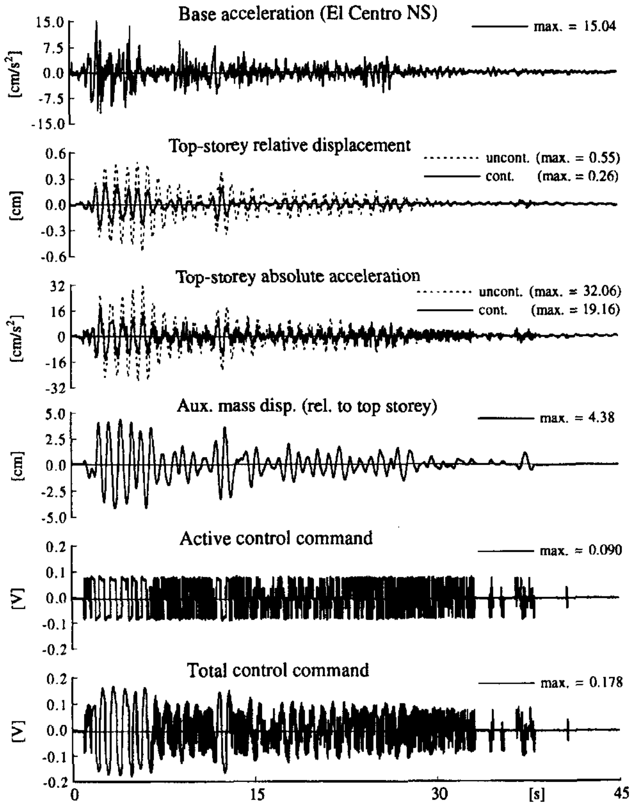


Figure 7. Experimental results under 15 cm/s^2 El Centro NS ground motion (dt = 10 ms)

Seismic excitations

Basic case. The time histories of non-control and active-control experimental results under 15 cm/s^2 El Centro NS ground motion record with 10 ms control time interval are shown in Figure 7. The displacement and acceleration responses of the top storey could be suppressed by about 50 and 40 per cent, respectively. The control commands have the unit of voltage with control gain 260 N/V. The high-frequency signals appeared in the control command time histories were caused by noise during the experiment; however, this high-frequency noise did not excite the auxiliary mass. It could be seen that the maximum control force for adjusting the auxiliary mass, u_{aux}, was approximately equal to the maximum active control force, u_{act}.

Using the recorded base acceleration and the identified properties of the actuator and the model structure, and taking into account the high-pass filter characteristics of the actuator's driver, analytical simulation was performed and the results are shown in Figure 8. It was clear that the analytical and experimental results agreed extremely well. Another simulation was carried out by assuming ideal condition of the actuator (without high-pass filter characteristics), and the results are presented in Figure 9. From these results it could be observed that the model responses show no significant differences with the experimental ones; therefore, we could conclude that for this experiment, the effect of high-pass filter on the control performance was negligible.

The experimental results for non-control and active-control condition under Taft EW ground motion record with peak acceleration of 15 cm/s^2 are presented in Figure 10. In this case, the displacement and acceleration responses of the top storey could be reduced to about half of the uncontrolled responses. Figure 11 shows the results of analytical simulation by taking into account the high-pass filter properties of the actuator's driver, which agreed very well with the experimental results for all responses except control commands and auxiliary mass acceleration due to the existence of high-frequency noise.

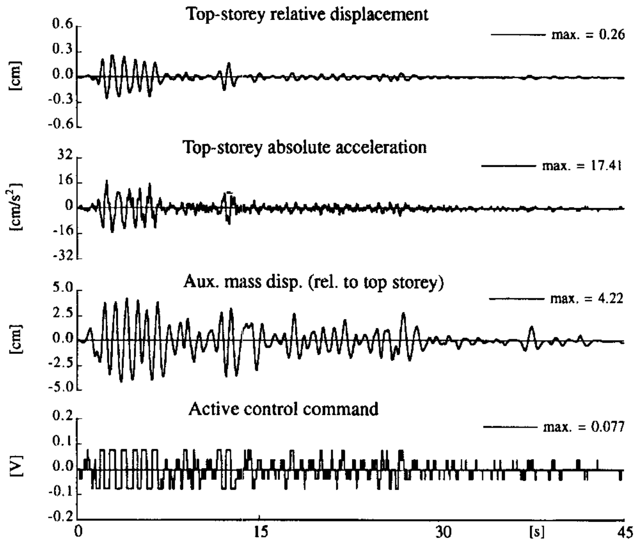


Figure 8. Analytical results under 15 cm/s^2 El Centro NS ground motion (with filter characteristics)

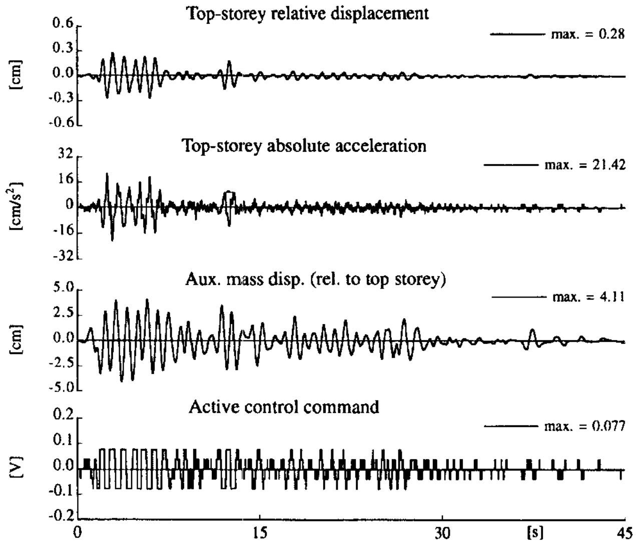


Figure 9. Analytical results under 15 cm/s^2 El Centro NS ground motion (without filter characteristics)

Higher excitation level. After confirming experimentally the control performance of the BFC algorithm under El Centro NS and Taft EW ground motion records and comparing it with the results of analytical simulations, we shall proceed to the next experiment which employed higher peak base acceleration. In this paper only the case of El Centro NS ground motion with peak acceleration of 30 cm/s^2 will be reported and discussed. It has been mentioned before that the maximum base acceleration was limited to 30 cm/s^2 due to the physical limitation of the actuator stroke length to 7 cm. The experimental results are shown in Figure 12. The employed active control force u_{act} for this test was also 10 and 20 N (two-level control force). With the same amount of u_{act}, the control performance was about 10 per cent less than in case of 15 cm/s^2 base acceleration; however, the stroke length of the auxiliary mass was only slightly increased compared to the case of 15 cm/s^2, which indicated one of the strong features of BFC. In case of other linear feedback control

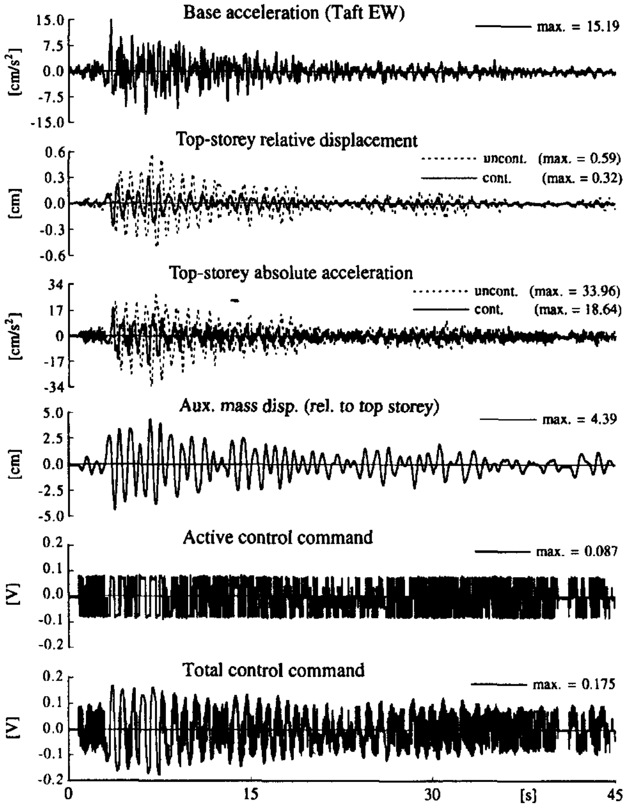


Figure 10. Experimental results under 15 cm/s^2 Taft EW ground motion (dt = 10 ms)

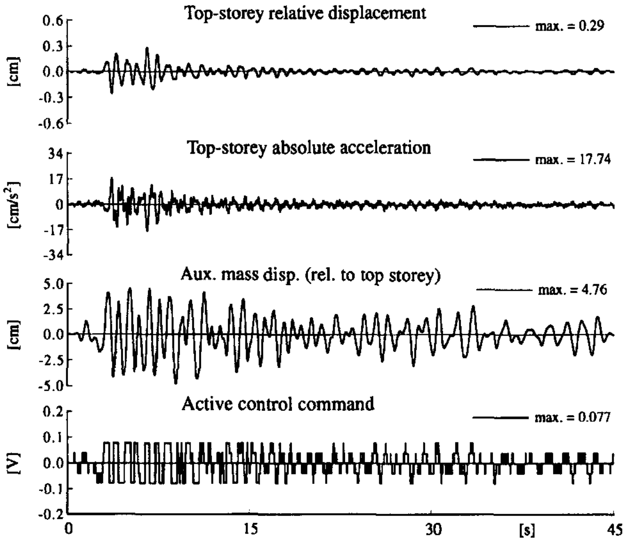


Figure 11. Analytical results under 15 cm/s^2 Taft EW ground motion (with filter characteristics)

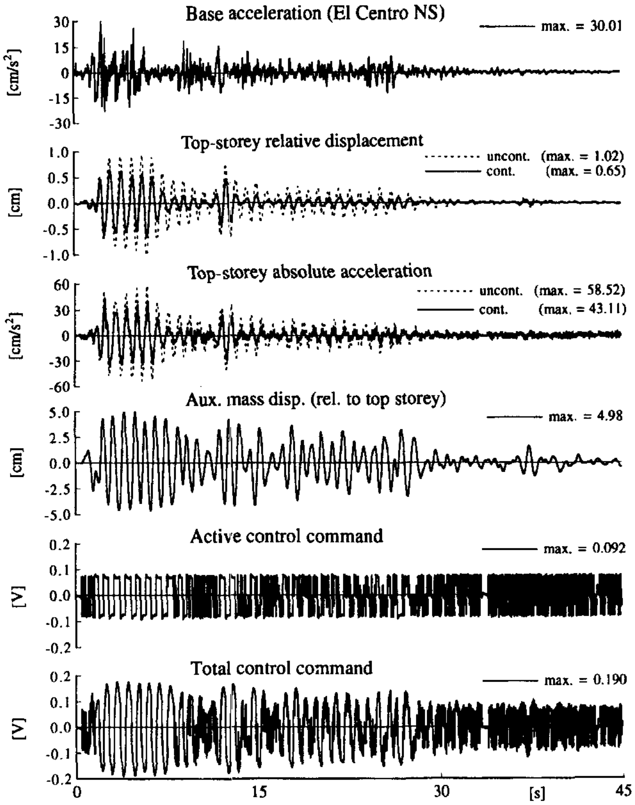


Figure 12. Experimental results under 30 cm/s^2 El Centro NS ground motion (dt = 10 ms)

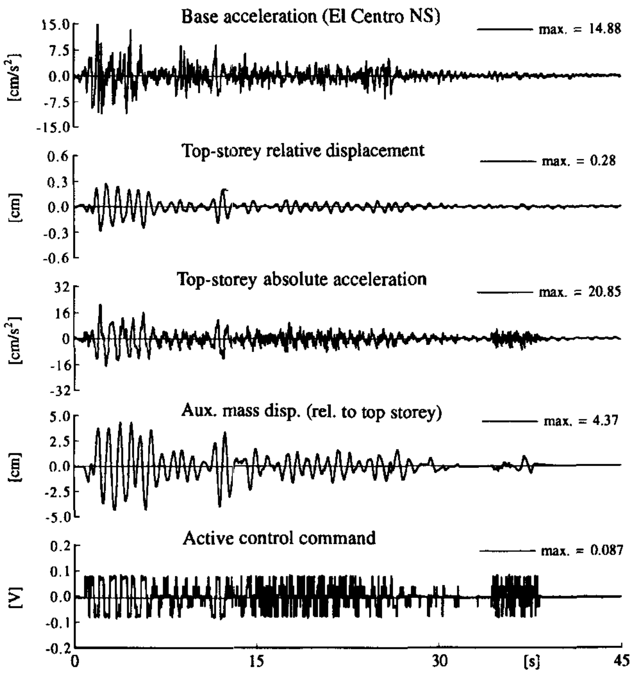


Figure 13. Experimental results under 15 cm/s^2 El Centro NS ground motion (dt = 30 ms)

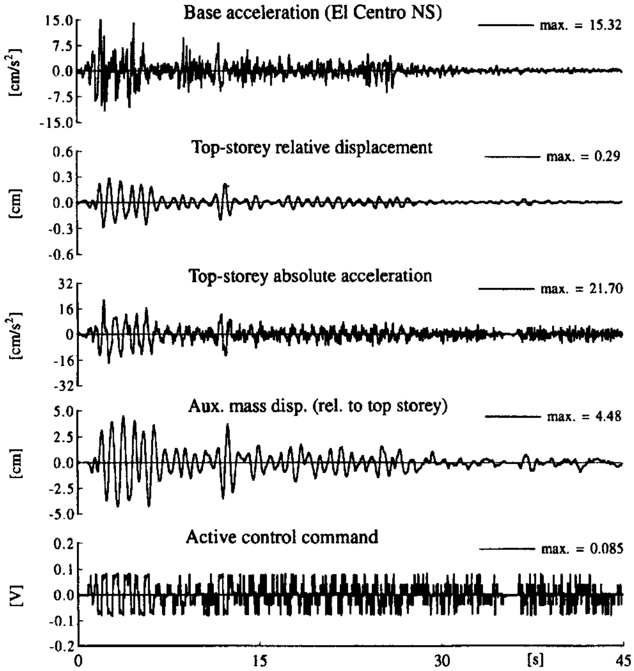


Figure 14. Experimental results under 15 cm/s^2 El Centro NS ground motion ($dt = 40$ ms)

methods, the stroke length, as well as the maximum active control force usually proportionally increase with the increase of base-excitation level.

Longer control time interval. As in the previous case, only the experimental results under El Centro ground motion will be presented here. The next series of active control experiments had the purpose of indirectly studying the effect of time delay on the BFC control performance. In the previously discussed experiments, the time interval dt was set to 10 ms, while for these experiments dt was changed to 30 and 40 ms, respectively. The time histories of the model and auxiliary mass responses as well as the control commands were shown in Figures 13 and 14, respectively, for $dt = 30$ and 40 ms. These time intervals were assumed long enough for practical implementation of an ATMD system to actual buildings. From these results it could be clearly seen that there was no negative effect of prolonged time step to the control performance. On the contrary, positive effect to the control command could be observed, that is, the high-frequency oscillations were significantly decreased.

CONCLUDING REMARKS

Following a brief review of the fundamental concept and main characteristics of the newly developed Bounded-Force Control method, the results of experimental verification of the method have been reported and discussed. Through analytical simulations, the validity of the experimental results have been confirmed.

It was demonstrated that even though the first-mode damping ratio of the model was quite high and the condition of the linear motor actuator was not ideal, the control performance of the BFC was significant and very stable, and to some extent is not sensitive to the distorted shape of the applied control force. The control performance is also consistent even under the increase of external disturbance level. It was also shown that time-delay effect did not destabilize or degrade the control performance of the BFC. This fact implied the viability of utilizing not only a linear motor, but also a servo-hydraulic motor as the actuator. Furthermore, it was observed that the excursion of the auxiliary mass (stroke length) was not proportionally increased with

the increase of base excitation level. These findings strengthened the conviction obtained through previous theoretical studies that the BFC is a potential method for response control of highrise building as well as other civil engineering structures under highly stochastic disturbances such as weak to strong earthquakes.

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